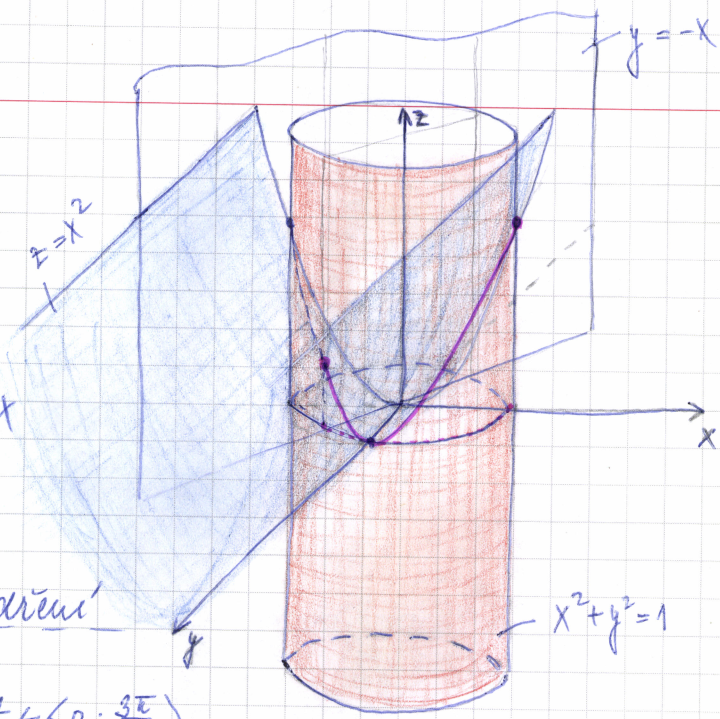
Zadání

$$\int_\gamma 2\cdot(z-y^2)\,xy\,ds$$

$$\gamma:\begin{cases} x^2+y^2=1 \\ z=x^2 \\ y\geq 0\,,\ y\geq -x\end{cases}$$

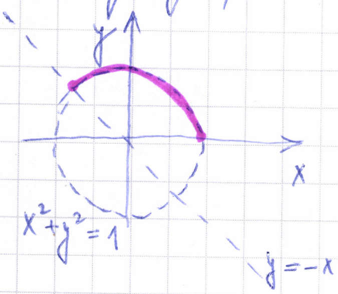


Parametrické vyjádření

$$x=\cos t \qquad y=\sin t \qquad z=\cos^2 t \qquad t\in\langle 0; \tfrac{3\pi}{4}\rangle$$

$$x' = -\sin t \qquad y' = \cos t \qquad z' = -2\cos t\sin t$$

Půdorysný průmět γ



$$\int_\gamma 2\cdot(z-y^2)\,xy\,ds = \int_0^{\frac{3\pi}{4}} 2\cdot(\cos^2 t-\sin^2 t)\cos t\sin t\sqrt{\sin^2 t+\cos^2 t+4\cos^2 t\sin^2 t}\,dt$$

$$=\frac{1}{2}\int_0^{\frac{3\pi}{4}} 2\cdot 2\cdot(\cos^2 t-\sin^2 t)\cos t\sin t\sqrt{\underbrace{1+4\cos^2 t\sin^2 t}_{u^2}}\,dt =$$

$$=\left|\begin{array}{l} 1+4\cos^2 t\sin^2 t = u^2 \\ -8\cos t\sin^3 t+8\cos^3 t\sin t\ dt = 2u\,du \\ 8\cos t\sin t(\cos^2 t-\sin^2 t)\,dt = 2u\,du \\ \begin{array}{c|c|c} t & 0 & \frac{3\pi}{4} \\ \hline u & 1 & \sqrt{2}\end{array}\end{array}\right| =$$

$$=\frac{1}{2}\int_1^{\sqrt{2}} u^2\,du = \frac{1}{6}\left[u^3\right]_1^{\sqrt{2}} = \frac{1}{6}\left(\sqrt{8}-1\right)$$