

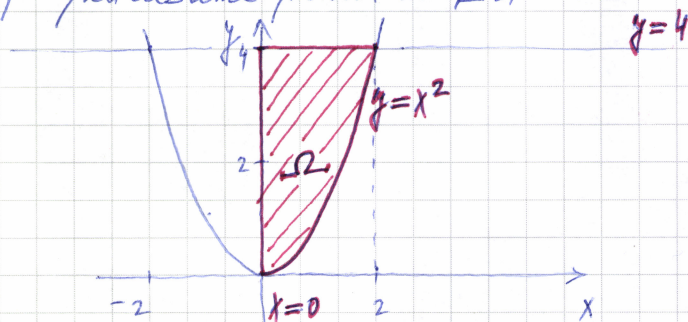
Mějme danou oblast $\Omega = \{[x,y] \in \mathbb{R}^2; y < 4, y > x^2, x > 0\}$

a) spočítejte integrál

$$\iint_{\Omega} x+y \, dx \, dy$$

b) načítejte těleso, jehož objem integrál reprezentuje

a) nakresleme množinu Ω



Meze integrace 1. způsob

$$0 \leq x \leq 2$$

$$x^2 \leq y \leq 4$$

2. způsob

$$0 \leq y \leq 4$$

$$0 \leq x \leq \sqrt{y}$$

$$\iint_{\Omega} x+y \, dx \, dy = \overset{1. \text{ způsob}}{=} \int_0^2 \left(\int_{x^2}^4 x+y \, dy \right) dx = \int_0^2 \left[xy + \frac{y^2}{2} \right]_{y=x^2}^4 dx =$$

$$\int_0^2 4x + 8 - \left(x^3 + \frac{x^4}{2} \right) dx = \left[\frac{4x^2}{2} + 8x - \frac{x^4}{4} - \frac{x^5}{10} \right]_0^2 = 20 - \frac{16}{5} = 16.8$$

$$\overset{2. \text{ způsob}}{=} \int_0^4 \left(\int_0^{\sqrt{y}} x+y \, dx \right) dy = \int_0^4 \left[\frac{x^2}{2} + yx \right]_{x=0}^{\sqrt{y}} dy =$$

$$= \int_0^4 \left(\frac{y}{2} + y\sqrt{y} \right) dy = \left[\frac{y^2}{4} + \frac{y^{5/2}}{5/2} \right]_0^4 = 4 + \frac{4 \cdot 16}{5} = 16.8$$

b)

