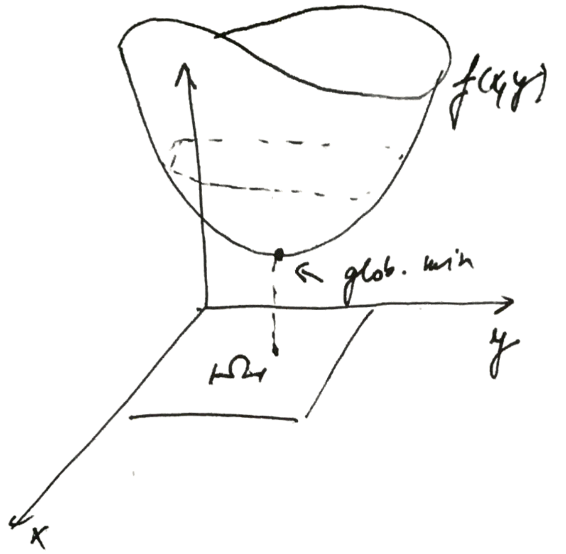
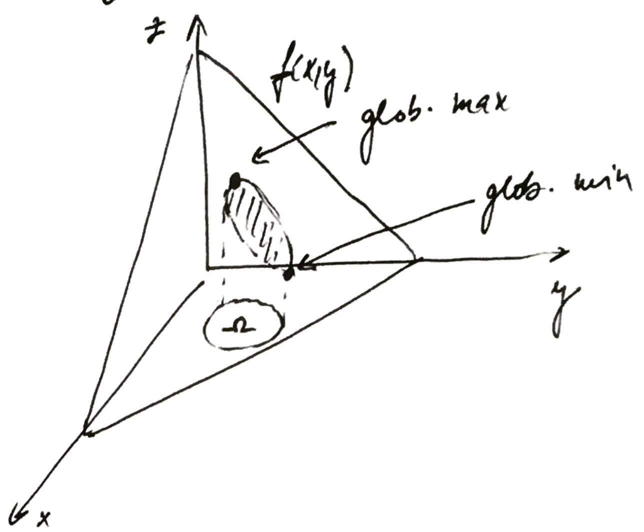
Funkce dvou proměnných – globální extrémy

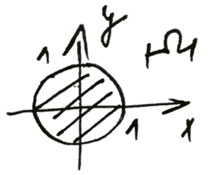




1) Hledáme lok. extr. uvnitř mn. Ω (ale není potřeba test pomocí druhých derivací)
2) Hledáme extrémy na hranici mn. Ω

Př: $f(x,y) = 6x^2 - 8x + 2y^2 - 5$

Ω: $x^2 + y^2 \leq 1$



$f_x(x,y) = 12x - 8 = 0 \Rightarrow 3x - 2 = 0 \Rightarrow x = \frac{2}{3}$

$f_y(x,y) = 4y = 0 \Rightarrow y = 0$

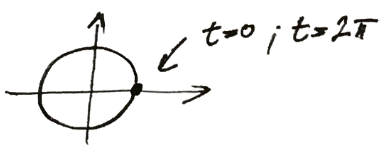
$A = [\frac{2}{3}; 0]$

? $A \in Ω$? $\quad (\frac{2}{3})^2 + (0)^2 \overset{?}{\leq} 1$

$\frac{4}{9} \leq 1$ ✓ $\Rightarrow$ Ano; $A \in Ω$

Γ ... hraniční křivka t.j. jednotková kružnice

$\boxed{\begin{aligned} x &= \cos t \\ y &= \sin t \end{aligned}}$ $\quad t \in \langle 0; 2\pi \rangle$



$F(t) = f(\cos t; \sin t) = 6\cos^2 t - 8\cos t + 2\sin^2 t - 5$

$F'(t) = 6 \cdot 2 \cdot \cos t \cdot (-\sin t) + 8\sin t + 2 \cdot 2 \sin t \cdot \cos t$

$= -12\cos t \sin t + 8\sin t + 4\cos t \sin t = \sin t \cdot (-12\cos t + 8 + 4\cos t)$

$= \sin t \cdot (8 - 8\cos t) = 0$

$\sin t \cdot (8 - 8\cos t) = 0$

$\cancel{8}\sin t \cdot \underbrace{(1 - \cos t)} = 0$

$t \in \langle 0; 2\pi \rangle$

$\sin t = 0 \qquad \cos t = 1$

$t = 0 \qquad t = 0$

$\boxed{t = \pi} \qquad t = 2\pi$

$t = 2\pi$

$\rightarrow B = [1; 0] \leftarrow \qquad t = 0 \; ; \; t = 2\pi$

$C = [-1; 0] \qquad t = \pi$

$f\left(\frac{2}{3}; 0\right) = -\frac{23}{3} \leftarrow$ glob. min

$f(1; 0) = -7$

$f(-1; 0) = 9 \leftarrow$ glob. max