Funkce dvou proměnných – Taylorův polynom

K čemu to je?

$$f(x,y) \approx \boxed{T(x,y)}$$

$\sin xy$ $\qquad\qquad$ $x^2y^4 - y^2 + 8$

Vzorec Taylorův polynom rozvitý v b. $[x_0,y_0]$

$$f(x,y) \approx T(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0) +$$

$$+ \frac{1}{2!}\Big(f_{xx}(x_0,y_0)(x-x_0)^2 + 2f_{xy}(x_0,y_0)(x-x_0)(y-y_0) + f_{yy}(x_0,y_0)(y-y_0)^2\Big) +$$

$$+ \frac{1}{3!}\Big(f_{xxx}(x_0,y_0)(x-x_0)^3 + 3f_{xxy}(x_0,y_0)(x-x_0)^2(y-y_0) + 3f_{yyx}(x_0,y_0)(x-x_0)(y-y_0)^2 +$$

$$+ f_{yyy}(x_0,y_0)(y-y_0)^3\Big)$$

Př $\quad f(x,y) = x^2e^y \quad$ v bodě $[-2,0]$ $\quad \boxed{n=2}$

$$\begin{aligned}
f(x,y) &= x^2e^y & f(-2,0) &= 4\\
f_x(x,y) &= 2xe^y & f_x(-2,0) &= -4\\
f_y(x,y) &= x^2e^y & f_y(-2,0) &= 4\\
f_{xx}(x,y) &= 2e^y & f_{xx}(-2,0) &= 2\\
f_{xy}(x,y) &= 2xe^y & f_{xy}(-2,0) &= -4\\
f_{yy}(x,y) &= x^2e^y & f_{yy}(-2,0) &= 4
\end{aligned}$$

$$x^2e^y \approx T(x,y) = 4 + (-4)(x-(-2)) + 4\cdot(y-0) + \frac{1}{2!}\Big(2(x-(-2))^2 + 2\cdot(-4)(x-(-2))(y-0) +$$

$$+ 4\cdot(y-0)^2\Big) = 4 - 4(x+2) + 4y + (x+2)^2 - 4(x+2)y + 2y^2$$