

Aplikace určitého integrálu

1) $L = ?$

$$x = e^t \sin t$$

$$y = e^t \cos t$$

$$t \in \langle 0; \frac{\pi}{2} \rangle$$

$$x' = e^t \sin t + e^t \cos t$$

$$y' = e^t \cos t + e^t (-\sin t) \\ = e^t \cos t - e^t \sin t$$

$$L = \int_{\alpha}^{\beta} \sqrt{(x')^2 + (y')^2} dt =$$

$$= \int_0^{\pi/2} \sqrt{\underbrace{e^{2t} \sin^2 t + 2e^{2t} \sin t \cos t + e^{2t} \cos^2 t}_{(x')^2} + \dots}$$

$$\dots + \underbrace{e^{2t} \cos^2 t - 2e^{2t} \sin t \cos t + e^{2t} \sin^2 t}_{(y')^2} dt =$$

$$= \int_0^{\pi/2} \sqrt{e^{2t} (\underbrace{\sin^2 t + \cos^2 t}_1 + \underbrace{\cos^2 t + \sin^2 t}_1)} dt =$$

$$= \int_0^{\pi/2} \sqrt{e^{2t} \cdot 2} dt = \int_0^{\pi/2} e^t \cdot \sqrt{2} dt = \sqrt{2} \int_0^{\pi/2} e^t dt =$$

$$\sqrt{2} [e^t]_0^{\pi/2} = \sqrt{2} \cdot (e^{\pi/2} - \underbrace{e^0}_1) = \underline{\underline{\sqrt{2} (e^{\pi/2} - 1)}} \quad \text{f'}$$

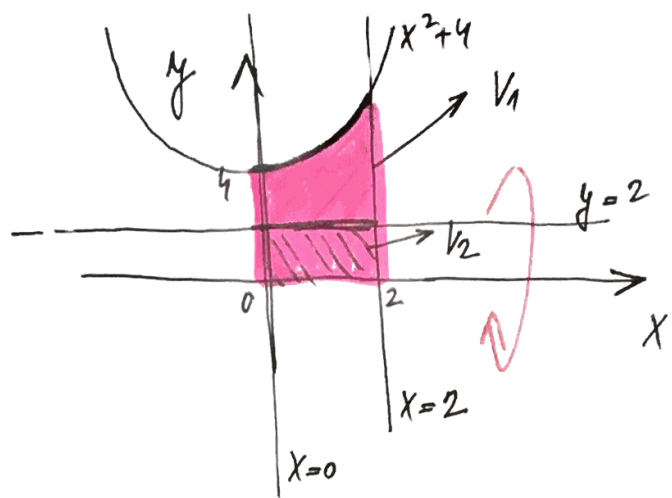
2) $V = ?$

$$y = x^2 + 4$$

$$y = 2$$

$$x = 0$$

$$x = 2$$



$$V = V_1 - V_2 =$$

$$= \pi \int_a^b f_1^2(x) dx - \pi \int_a^b f_2^2(x) dx =$$

$$= \pi \int_0^2 (x^2 + 4)^2 dx - \pi \int_0^2 (2)^2 dx = \pi \int_0^2 x^4 + 8x^2 + 16 dx - \pi \int_0^2 4 dx =$$

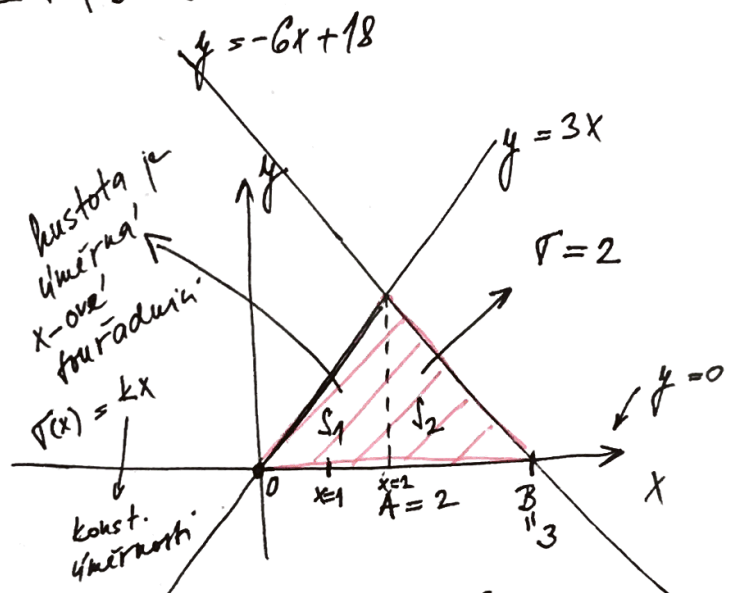
$$= \pi \left[\frac{x^5}{5} + \frac{8x^3}{3} + 16x \right]_0^2 - \pi [4x]_0^2 = \pi \left(\frac{32}{5} + \frac{64}{3} + 32 \right) - \pi \cdot 8 =$$

3) $V = ?$, $\text{těžiště} = ?$

$$y = 3x$$

$$y = -6x + 18$$

$$y = 0$$



$$V = \int_0^2 3x dx + \int_2^3 (-6x + 18) dx$$

$$m = \int_a^b r(x) \cdot f(x) dx =$$

$$\int_0^2 kx \cdot 3x dx + \int_2^3 2 \cdot (-6x + 18) dx$$

$$S_x = \frac{1}{2} \int_a^b r(x) f^2(x) dx =$$

$$= \frac{1}{2} \left(\int_0^2 kx \cdot (3x)^2 dx + \int_2^3 2 \cdot (-6x + 18)^2 dx \right)$$

$$S_y = \int_a^b r(x) \cdot x \cdot f(x) dx = \int_0^2 kx \cdot x \cdot 3x dx + \int_2^3 2 \cdot x \cdot (-6x + 18) dx$$

$$A = ?$$

$$y = 3x$$

$$y = -6x + 18$$

$$3x = -6x + 18$$

$$9x = 18$$

$$x = 2$$

$$B = ?$$

$$y = -6x + 18$$

$$y = 0$$

$$-6x + 18 = 0$$

$$6x = 18$$

$$x = 3$$