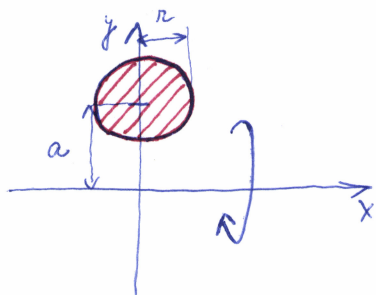


Annuloid

Těleso vzniklé rotací kolem osy x kružnice

$$x^2 + (y-a)^2 = r^2 \quad a > r$$



$$V = \pi \int_{\alpha}^{\beta} r^2(t) |\varphi'(t)| dt$$

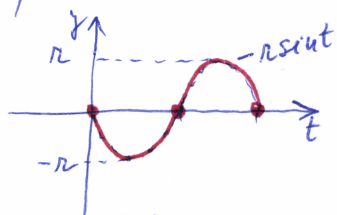
$$r(t) \geq 0, \quad \varphi'(t) \neq 0, \quad \forall t \in (\alpha, \beta)$$

$$x = r \cdot \cos t = \varphi(t)$$

$$y = r \cdot \sin t + a = r(t) \quad t \in \langle 0, 2\pi \rangle$$

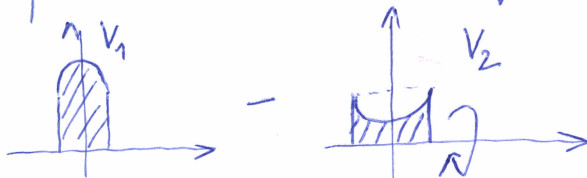
$$r(t) > 0$$

$$\varphi'(t) = -r \cdot \sin t$$



\Rightarrow objem může spočítat jako jeden integrál

$$V = V_1 - V_2$$



$$V = \pi \int_0^{\pi} (r \cdot \sin t + a)^2 \cdot \underbrace{|-r \sin t|}_{\text{záporné}} dt - \pi \int_{\pi}^{2\pi} (r \cdot \sin t + a)^2 \cdot \underbrace{|-r \sin t|}_{\text{kladné}} dt$$

$$= \pi \int_0^{\pi} r^3 \sin^3 t + 2ar^2 \sin^2 t + na^2 \sin t dt + \pi \int_{\pi}^{2\pi} r^3 \sin^3 t + 2ar^2 \sin^2 t + na^2 \sin t dt$$

$$= \pi \int_0^{2\pi} \underbrace{r^3 \sin^3 t}_{\text{periodická}} + 2ar^2 \sin^2 t + \underbrace{na^2 \sin t}_{\text{periodická}} dt = \pi \cdot 2ar^2 \cdot \int_0^{2\pi} \sin^2 t dt =$$

$$= 2\pi ar^2 \int_0^{2\pi} \frac{1 - \cos 2t}{2} dt = \pi ar^2 \left[t - \frac{1}{2} \sin 2t \right]_0^{2\pi} = \underline{\underline{2\pi^2 ar^2}}$$

$$P = \int_{\alpha}^{\beta} \psi(t) \cdot \sqrt{[\varphi'(t)]^2 + [\psi'(t)]^2} dt$$

$$\psi(t) \geq 0 \quad t \in (\alpha, \beta)$$

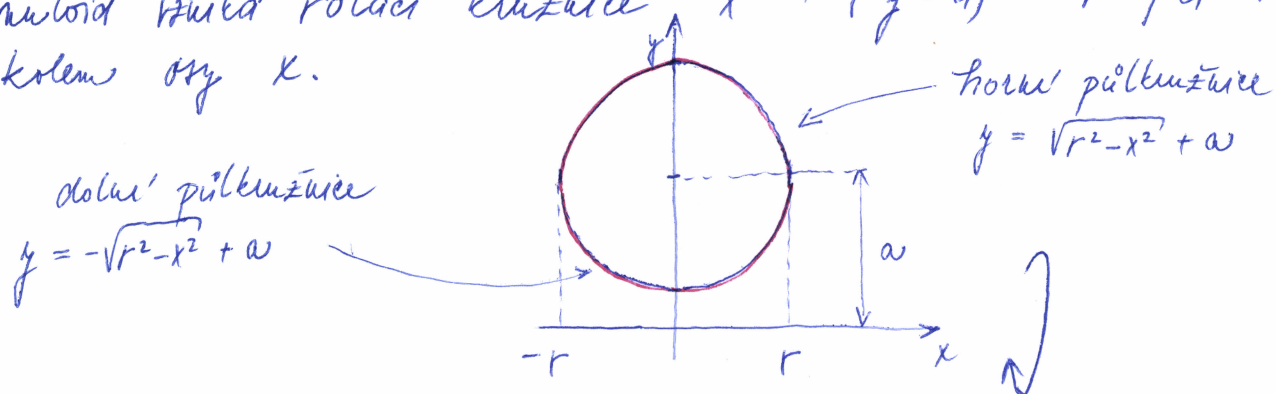
$$P = \int_0^{2\pi} (r \cdot \sin t + a) \cdot \sqrt{(r^2 \sin t)^2 + (r \cos t)^2} dt$$

$$= \int_0^{2\pi} r^2 \sin t + ar dt = \underbrace{2\pi r^2 [-\cos t]_0^{2\pi}}_0 + 2\pi ar [t]_0^{2\pi}$$

$$= \underline{\underline{4\pi^2 ar}}$$

Wěte objem anuloidu a povrch anuloidové plochy.

Anuloid vzniká rotací kružnice $x^2 + (y-a)^2 = r^2$, $a > r$, kolem osy x .



$$V = V_1 - V_2 = \pi \int_{-r}^r (\sqrt{r^2 - x^2} + a)^2 dx - \pi \int_{-r}^r (-\sqrt{r^2 - x^2} + a)^2 dx =$$

$$= \pi \int_{-r}^r (r^2 - x^2 + 2a\sqrt{r^2 - x^2} + a^2 - (r^2 - x^2 - 2a\sqrt{r^2 - x^2} + a^2)) dx =$$

$$= 4a\pi \int_{-r}^r \sqrt{r^2 - x^2} dx = \left| \begin{array}{l} x = r \sin t \\ dx = r \cos t dt \\ \begin{array}{c|c|c} x & -r & r \\ \hline t & -\frac{\pi}{2} & \frac{\pi}{2} \end{array} \end{array} \right| =$$

$$= 4ar\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{r^2 - r^2 \sin^2 t} \cos t dt = 4ar^2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2 t dt =$$

$$= 4ar^2\pi \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} (1 + \cos 2t) dt = 2ar^2\pi \left[t + \frac{1}{2} \sin 2t \right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \underline{\underline{2ar^2\pi^2}}$$

$$P = P_1 + P_2 = 2\pi \int_{-r}^r (\sqrt{r^2 - x^2} + a) \cdot \sqrt{1 + \frac{x^2}{r^2 - x^2}} dx +$$

$$+ 2\pi \int_{-r}^r (-\sqrt{r^2 - x^2} + a) \cdot \sqrt{1 + \frac{x^2}{r^2 - x^2}} dx =$$

$$= 4\pi ar \int_{-r}^r \frac{1}{\sqrt{r^2 - x^2}} dx = \left| \begin{array}{l} x = r \sin t \\ dx = r \cos t dt \\ \begin{array}{c|c|c} x & -r & r \\ \hline t & -\frac{\pi}{2} & \frac{\pi}{2} \end{array} \end{array} \right| =$$

$$= 4\pi ar \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1 dt = \underline{\underline{4\pi^2 ar}}$$