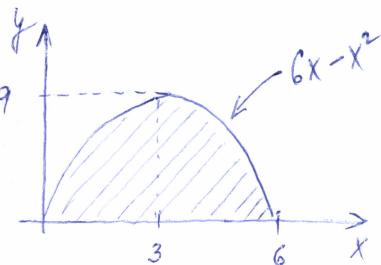
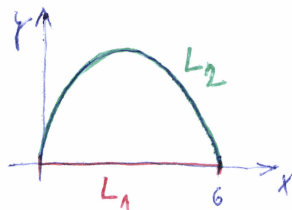


Mějme dan obzobec ohraničený křivkami
 $y = 6x - x^2$ a $y = 0$



a) Vypočítejte jeho obvod a obsah

$$L = L_1 + L_2$$



$$L_1 = 6$$

$$L_2 = \int_0^6 \sqrt{1 + [f'(x)]^2} dx = \int_0^6 \sqrt{1 + (6-2x)^2} dx = \left| \begin{array}{l} 6-2x = t \\ -2dx = dt \\ dx = -\frac{1}{2}dt \end{array} \right| \begin{array}{c|c|c} x & 0 & 6 \\ \hline t & 6 & -6 \end{array}$$

$$= -\frac{1}{2} \int_6^{-6} \sqrt{1+t^2} dt = \frac{1}{2} \int_{-6}^6 \sqrt{1+t^2} dt = \left| \begin{array}{l} u = \sqrt{1+t^2} \\ u' = \frac{t}{\sqrt{1+t^2}} \end{array} \right|$$

$$= \frac{1}{2} \left([t\sqrt{1+t^2}]_{-6}^6 - \int_{-6}^6 \frac{t^2+1-1}{\sqrt{1+t^2}} dt \right) =$$

$$= 6\sqrt{36} - \frac{1}{2} \int_{-6}^6 \sqrt{1+t^2} dt + \frac{1}{2} \int_{-6}^6 \frac{1}{\sqrt{1+t^2}} dt =$$

$$= 6\sqrt{36} - \frac{1}{2} \int_{-6}^6 \sqrt{1+t^2} dt + \frac{1}{2} \left[\ln |t + \sqrt{1+t^2}| \right]_{-6}^6 =$$

$$= 6\sqrt{36} - \frac{1}{2} \int_{-6}^6 \sqrt{1+t^2} dt + \frac{1}{2} \ln(6 + \sqrt{37}) - \frac{1}{2} \ln(-6 + \sqrt{37})$$

\Downarrow

$$+\frac{1}{2} I = 6\sqrt{36} + \frac{1}{2} \ln(6 + \sqrt{37}) - \frac{1}{2} \ln(-6 + \sqrt{37}) - \frac{1}{2} I$$

$$I = 6\sqrt{36} + \frac{1}{2} \ln(6 + \sqrt{37}) - \frac{1}{2} \ln(-6 + \sqrt{37})$$

$$\Rightarrow L_2 = \frac{1}{2} I = 3\sqrt{36} + \frac{1}{4} (\ln(6 + \sqrt{37}) - \ln(-6 + \sqrt{37})) \doteq 19.5$$

$$L = \underline{25.5} \text{ j}$$

$$S = \int_0^6 6x - x^2 dx = \left[\frac{6x^2}{2} - \frac{x^3}{3} \right]_0^6 = \underline{36} \text{ j}^2$$

- b) Vypočítejte souřadnice těžiště vte - G , že těleso je homogenní

$$\Rightarrow \text{hustota } \rho(x) = c$$

$$\text{těžiště } T = [T_x, T_y], \text{ víme, že } T_x = 3$$

$$T_y = \frac{J_x}{m}$$

$$m = \int_a^b \rho(x) \cdot f(x) dx = \int_0^6 c \cdot (6x - x^2) dx = c \left[\frac{6x^2}{2} - \frac{x^3}{3} \right]_0^6 = 36 \cdot c$$

$$J_x = \frac{1}{2} \int_a^b \rho(x) \cdot f^2(x) dx = \frac{1}{2} \int_0^6 c \cdot (6x - x^2)^2 dx =$$

$$= \frac{1}{2} c \int_0^6 (36x^2 - 12x^3 + x^4) dx = \frac{1}{2} c \left[\frac{36x^3}{3} - \frac{12x^4}{4} + \frac{x^5}{5} \right]_0^6 = 129.6$$

$$T_y = \frac{129.6 \cdot c}{36 \cdot c} = 3.6$$

- hustota tělesa je přímo úměrná x -ové souřadnici

$$\Rightarrow \text{hustota } \rho(x) = c \cdot x$$

$$m = \int_0^6 c \cdot x \cdot (6x - x^2) dx = c \int_0^6 (6x^2 - x^3) dx = c \left[\frac{6x^3}{3} - \frac{x^4}{4} \right]_0^6 = 108c$$

$$J_x = \frac{1}{2} \int_0^6 c \cdot x \cdot (6x - x^2)^2 dx = \frac{c}{2} \int_0^6 (36x^3 - 12x^4 + x^5) dx = \frac{c}{2} \left[\frac{36x^4}{4} - \frac{12x^5}{5} + \frac{x^6}{6} \right]_0^6 = 388.8c$$

$$J_y = \int_a^b \rho(x) \cdot x \cdot f(x) dx = \int_0^6 c \cdot x \cdot (6x - x^2) dx = c \int_0^6 (6x^2 - x^3) dx = c \left[\frac{6x^3}{3} - \frac{x^4}{4} \right]_0^6 = 388.8c$$

$$T_x = \frac{J_y}{m} = 3.6 \quad T_y = \frac{J_x}{m} = 3.6$$

- těleso se skládá ze dvou homogenních částí. Hustota levé poloviny je dvakrát větší než hustota pravé poloviny.

$$m = \int_0^3 2c \cdot (6x - x^2) dx + \int_3^6 c \cdot (6x - x^2) dx = 54c$$

$$J_x = \frac{1}{2} \left(\int_0^3 2c (6x - x^2)^2 dx + \int_3^6 c (6x - x^2)^2 dx \right) = 191.4c$$

$$J_y = \int_0^3 2c \cdot x \cdot (6x - x^2) dx + \int_3^6 c \cdot x \cdot (6x - x^2) dx = 141.75c$$

$$\Rightarrow T_x = 2.625 \quad T_y = 3.6$$

