Eliminační metodou vyřešte počáteční problém

$$y_1'' = -4y_1 + 2y_2 \qquad y_1(0) = 0 \qquad y_1'(0) = 0.2$$
$$y_2'' = 2y_1 - 2y_2 \qquad y_2(0) = 0 \qquad y_2'(0) = 0$$

<u>Řešení:</u>

- Ze soustavy eliminujeme neznámou y_2

$$\left.\begin{aligned} y_1'' &= -4y_1 + 2y_2 \\ y_2'' &= 2y_1 - 2y_2 \end{aligned}\right\} + \quad \Rightarrow \quad y_2 = \frac{1}{2}y_1'' + 2y_1 \quad \Rightarrow \quad y_2'' = \frac{1}{2}y_1^{(4)} + 2y_1''$$

$$y_1'' + y_2'' = -2y_1$$
$$y_1'' + \frac{1}{2}y_1^{(4)} + 2y_1'' = -2y_1 \quad \Longleftrightarrow \quad \boxed{y_1^{(4)} + 6y_1'' + 4y_1 = 0}$$ ... homogenní lin. rce 4. řádu

- Vyřešíme LODR4 $y_1^{(4)} + 6y_1'' + 4y_1 = 0$

Charakteristická rovnice : $\lambda^4 + 6\lambda^2 + 4 = 0 \qquad \lambda^2 = L$

$$L^2 + 6L + 4 = 0$$

$$\Rightarrow L_{1,2} = \frac{-6 \pm \sqrt{20}}{2} = -3 \pm \sqrt{5} \quad \Rightarrow \quad \lambda_{1,2} = \sqrt{3+\sqrt{5}}\, i = \alpha i$$
$$\lambda_{3,4} = \sqrt{3-\sqrt{5}}\, i = \beta i$$

$$\Rightarrow \boxed{y_1 = c_1 \cos\alpha x + c_2 \sin\alpha x + c_3 \cos\beta x + c_4 \sin\beta x}$$

- Dopočítáme y_2 ; víme, že $y_2 = \frac{1}{2}y_1'' + 2y_1$

$$y_1' = -c_1\alpha \sin\alpha x + c_2\alpha\cos\alpha x - c_3\beta\sin\beta x + c_4\beta\cos\beta x$$
$$y_1'' = -c_1\alpha^2\cos\alpha x - c_2\alpha^2\sin\alpha x - c_3\beta^2\cos\beta x - c_4\beta^2\sin\beta x$$

$$\Rightarrow \boxed{y_2 = c_1\left(2 - \frac{\alpha^2}{2}\right)\cos\alpha x + c_2\left(2-\frac{\alpha^2}{2}\right)\sin\alpha x + c_3\left(2-\frac{\beta^2}{2}\right)\cos\beta x + c_4\left(2-\frac{\beta^2}{2}\right)\sin\beta x}$$

- Z počátečních podmínek určíme konstanty c_1, c_2, c_3, c_4

$$y_1(0) = 0: \quad c_1 + c_3 = 0$$
$$y_1'(0) = 0.2: \quad c_2\alpha + c_4\beta = 0.2$$
$$y_2(0) = 0: \quad c_1\left(2-\frac{\alpha^2}{2}\right) + c_3\left(2-\frac{\beta^2}{2}\right) = 0$$
$$y_2'(0) = 0: \quad c_2\alpha\left(2-\frac{\alpha^2}{2}\right) + c_4\beta\left(2-\frac{\beta^2}{2}\right) = 0$$

$$\Rightarrow \boxed{\begin{aligned} c_1 &= 0 \\ c_2 &= \left(0.2 - \frac{0.2\left(2-\frac{\alpha^2}{2}\right)}{\left(\frac{\beta^2}{2}-\frac{\alpha^2}{2}\right)}\right)\cdot\frac{1}{\alpha} \doteq 0.6325 \\ c_3 &= 0 \\ c_4 &= \frac{0.2\left(2-\frac{\alpha^2}{2}\right)}{\beta\left(\frac{\beta^2}{2}-\frac{\alpha^2}{2}\right)} \doteq 0.6325 \end{aligned}}$$